Adaptive, Semi-Supervised Consensus Model for Multi-Criteria Large Group Decision Making in a Linguistic Setting

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***Abstract*—In this paper we investigate the consensus reaching problem for Large Group Multi-Criteria Decision Making (MCLGDM). We present an adaptive, semi-supervised consensus model for MCLGDM problems with preferences expressed as Comparative Linguistic Expressions. Specifically, our work introduces an adaptive, semi-supervised feedback mechanism that, depending on the positions of decision makers' preferences and their level of uncertainty caused by hesitancy, requests human supervision to modify their preferences or updates them automatically. The proposed consensus model effectively handles large amounts of linguistic-natured information in consensus processes involving large groups. The methodology is illustrated and experimentally validated through a MCLGDM problem for candidate assessment in recruiting processes. Likewise, a theoretical comparison with similar works is provided.**

I. Introduction

Decision making plays a central role in daily mankind activities. Real-life decisions must be often made under uncertainty caused by the vagueness and imprecision of the decision maker (DM) to assess alternatives. Fuzzy set theory and the fuzzy linguistic approach [1, 2, 3, 4] have long been used as powerful tools for preference modeling under uncertainty in diverse decision frameworks, including (i) *Group Decision Making* (GDM) problems, where multiple DMs with diverse points of view attempt to make a common decision; and (ii) *Multi-Criteria Decision Making* problems, in which alternatives are evaluated according to several criteria [5]. Recently, the concept of Hesitant Fuzzy Linguistic Term Sets (HFLTSs) [6] has attained special attention to represent (and deal with) uncertain linguistic preferential information [7, 8] in a variety of the above mentioned decision frameworks.

The growing need for highly accepted group decisions (not guaranteed by classical GDM methods) has led a research shift towards consensus-based GDM approaches [9], aimed at bringing DMs preferences closer to each other before making a decision. Consensus has become a prominent research subject within GDM in the last decades [10, 11, 12, 13], with recent notable efforts on extending consensus approaches to problems involving considerably larger groups of DMs [14, 15, 16].

This work studies consensus reaching in *Multi-Criteria Large Group Decision Making* (MCLGDM) problems where DMs may have vague knowledge about the problem at hand, hence they assess alternatives under several criteria using Comparative Linguistic Expressions (CLEs), e.g. "low","between low and medium", "at least medium". Closely related to the concept of HFLTS, a CLE enables higher flexibility and expressiveness in situations when a DM hesitates among several linguistic terms. The study of consensus approaches in decision problems based on CLEs is still scarce to date, with some aspects requiring further investigation:

1) The uncertainty (given by vagueness and hesitancy) inherent to CLE-based preferences should be taken into account when measuring the level of consensus in the group, and minimized before reaching consensus, so as to ensure that not only highly accepted, but also reliable collective decisions are made.
2) The temporal cost invested by DMs in revising their preferences has proved to escalate when the size of the group increases [15]. To the best of our knowledge, existing consensus models using hesitant linguistic information, have not taken this aspect into consideration as of yet.

This paper presents an adaptive, semi-supervised consensus model for MCLGDM problems in which DMs use CLEs to assess alternatives. The novel contribution of our work is threefold. Firstly, we introduce an adaptive, semi-supervised rule-based feedback mechanism for reaching consensus, which depending on the characteristics and hesitancy in DMs preferences, requests DMs supervision to modify their preferences or updates them automatically. This not only reduces the above mentioned cost, but also preserves the DM sovereignty to some extent. Secondly, the feedback mechanism incorporates rules focused on minimizing the uncertainty in DMs assessments. Thirdly, although assessments are transformed into HFLTS to apply computational processes such as preference aggregation, preferential information exchanged with DMs is always represented by CLEs, to facilitate their understanding.

The remainder of this paper is organised as follows. Section

2 formalises the target MCLGDM framework and revisits preliminaries on consensus reaching, CLEs and HFLTS. Section 3 presents the adaptive, semi-supervised consensus model for MCLGDM problems with CLEs. An application example in an academic recruitment problem, along with a theoretical comparison with similar methods, are presented in Section 4. Finally, conclusions are drawn in Section 5.

II. Background

A. MCLGDM Framework

GDM problems are characterized by the participation of multiple DMs in a decision problem aimed at selecting the best alternative/s from a finite set of them [5, 9]. The rapid ICT[1] advances are facilitating the participation of larger groups in these problems. Thus, LGDM is attracting the attention of researchers [14, 15, 16], in response to the growing necessity to undertake decision problems involving dozens or hundreds of DMs. In this work we adopt the notion of LGDM from [17], as a problem involving more than 11 DMs. Likewise, we concentrate on LGDM problems in which DMs evaluate alternatives in accordance to multiple criteria [5, 18], i.e. MCLGDM problems. Thus, the MCLGDM framework considered in this paper is formally characterised by the following elements:

- A finite set $X=\{x_1,\dots,x_n\}$, $(n\geq 2)$ of *alternatives*.
- A group, $E=\{e_1,\dots,e_m\}$, $(m\gg 2)$ of DMs or *experts* who express their opinions on the alternatives in X.
- A DM $e_i\in E$ $(i=1,\dots,m)$, must assess each alternative $x_j\in X$ $(j=1,\dots,n)$ according to a finite set of *criteria*, $C=\{c_1,\dots,c_l\}$, $(l\geq 1)$.

Each DM provides her/his opinions over $X\times C$ by means of an $n\times l$ evaluation matrix:

$$P_i=\begin{pmatrix} p_i^{11} & \dots & p_i^{1l} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \dots & p_i^{nl} \end{pmatrix}_{n\times l}$$

Each assessment $p_i^{jk}\in D$, expressed in an information domain D, indicates the degree to which x_j satisfies the k-th criterion $(k=1,\dots,l)$, according to e_i. Some information domains classically utilized for preference modeling include [19]: numerical, interval-valued and linguistic. Our focus is on MCLGDM problems in a linguistic setting, i.e. assessments are qualitatively expressed as linguistic terms $s_h\in S$, $(h=0,\dots,g)$, with $S=\{s_0,\dots,s_g\}$ a linguistic term set of granularity g [2, 3, 4]. An example of linguistic term with $g=4$ is, $S=\{s_0:Very\ Low(VL), s_1:Low(L), s_2:Medium(M), s_3:High(H), s_4:Very\ High(VH)\}$

Linguistic preference modeling approaches and computational models, such as the paradigm of *Computing with Words* [20], have been recently extended to allow for richer linguistic expressions that are much closer to human natural language, by introducing the concept of HFLTS and CLE [6].

[1]ICT: Information and Communication Technologies

B. Hesitant Fuzzy Linguistic Preferential Information

In linguistic decision making contexts, DMs may sometimes hesitate in choosing a single linguistic term $s_h\in S$ to provide an assessment. CLEs were introduced to allow DMs assessing decision information in such situations of hesitancy, in a human friendly fashion. They are closely related to the concept of HFLTS, originally introduced in [6] as follows.

Definition 1. Let $S=\{s_0,s_1,\dots,s_g\}$ be a linguistic term set with granularity g. An HFLTS $H_S=\{s_L,s_{L+1},\dots,s_U\}$ on S (with $0\leq L\leq U\leq g$), is an ordered finite non-empty subset of consecutive linguistic terms in S. The score or central value of H_S is given by $T(H_S)=\sum_{s_h\in H_S} h/|H_S|$ where $T(H_S)\in[0,g]$, $|H_S|\geq 1$, and:

$$\begin{cases} T(H_S) &= L=U \quad if\ |H_S|=1 \\ T(H_S) &= \frac{\sum_{s_h\in H_S} h}{U-L+1} \quad if\ |H_S|>1 \end{cases} \tag{1}$$

The following comparison rules were introduced for any two HFLTSs H_S^1, H_S^2 [6, 8] (with $H_S^1\succ H_S^2$ indicating that H_S^1 is preferred over H_S^2):

1) If $T(H_S^1)<T(H_S^2)$, then $H_S^1\prec H_S^2$.
2) If $T(H_S^1)>T(H_S^2)$, then $H_S^1\succ H_S^2$.
3) If $T(H_S^1)=T(H_S^2)$, then:
   a) If $|H_S^1|<|H_S^2|$ then $H_S^1\succ H_S^2$.
   b) If $|H_S^1|>|H_S^2|$ then $H_S^1\prec H_S^2$.
   c) If $|H_S^1|=|H_S^2|$ then $H_S^1=H_S^2$.

Moreover, the aggregation problem for HFLTSs has been actively investigated, with various aggregation operators proposed by diverse authors [7, 8, 18]. For instance, the works in [7, 8] introduced aggregation operators over HFLTSs whose results are represented as possibility[2] distributions over S, $\pi=\{\langle s_h,\beta_h\rangle\}$, with $\beta_h\in]0,1]$ representing the degree of possibility of H_S being s_h, and $\sum_h\beta_h=1$. Intuitively, any HFLTS H_S can be expressed as a uniform possibility distribution $\pi=\cup_{s_h\in H_S}\{\langle s_h,1/|H_S|\rangle\}$. Despite the operators in [7, 8] avoid any loss of information, the resulting possibility distribution is in general not uniform.

For the sake of expressiveness and accuracy in computations, an approach to transform a CLE into its equivalent HFLTS was introduced in [6]. In their approach, four types of linguistic expressions can be generated based on the use of a context-free grammar: (i) s_h, (ii), $at\ least\ s_L$ (iii) $at\ most\ s_U$, and (iv) $between\ s_L\ and\ s_U$.

Definition 2. Let $\mathcal{E}$ be a function that transforms a CLE generated by a context-free grammar, into a HFLTS H_S on S. A CLE is converted into its equivalent HFLTS by means of the following transformations:

- $\mathcal{E}(s_h)=\{s_h\}$.
- $\mathcal{E}($"*at least* s_L"$)=\{s_h|s_h\in S \text{ and } h\geq L\}$

[2]The term "possibility" (rather than "probability") is commonly adopted in related literature within this context. The non-zero values in possibility distributions over S shall not be interpreted under a frequentist probability point of view, but rather as the plausibility or state of knowledge of an agent (DM) about an actual state of affairs (assessments).

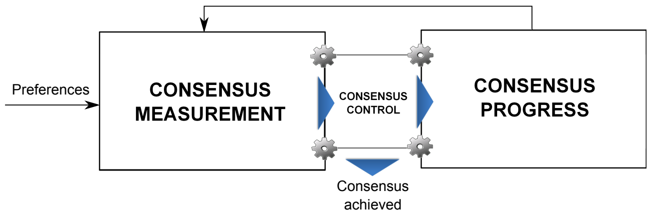

Fig. 1. General CRP scheme.

- $\mathcal{E}$(*"at most* s_U*"*)$=\{s_h|s_h \in S$ and $h \leq U\}$
- $\mathcal{E}$(*"between* s_L *and* s_U*"*)$=\{s_h|s_h \in S$ and $L \leq h \leq U\}$

In this work, motivated by the need for providing DMs with understandable preferential information anytime throughout a consensus building session, we introduce an inverse transformation function from a HFLTS to a CLE.

Definition 3. Let $H_S = \{s_L, \ldots, s_U\}$ be a HFLTS in $S = \{s_0, \ldots, s_g\}$. A transformation function Λ that converts H_S to its equivalent CLE is defined as follows:

$$\Lambda(H_S)=\begin{cases} s_L, & \text{if } L=U,\\ \textit{at least } s_L, & \text{if } 0<L<U=g,\\ \textit{at most } s_U, & \text{if } 0=L<U<g,\\ \textit{between } s_L \textit{ and } s_U, & \text{otherwise.}\end{cases} \tag{2}$$

Example 1. Let $S = \{s_0 : VL,\ s_1 : L,\ s_2 : M,\ s_3 : H,\ s_4 : VH\}$ be a linguistic term set with granularity $g = 4$, and $H_S = \{VL, L, M\}$ a HFLTS on S. Based on Definition 1, its score value is $T(H_S) = 1$ and its cardinality is $|H_S| = 3$. The associated possibility distribution for H_S is $\pi = \{\langle s_0, 0.33\rangle, \langle s_1, 0.33\rangle, \langle s_2, 0.33\rangle\}$, and its equivalent CLE is $\Lambda(H_S) =$*"at most Medium"*.

C. Consensus Approaches in GDM

The classical alternative selection process to solve GDM problems is composed of two phases [1]: (i) *aggregation*, in which preferences of DMs are combined by using an aggregation operator; and (ii) *exploitation*, i.e. applying a selection criterion to identify the alternative/s to be chosen as the solution for the GDM problem.

Classical GDM approaches do not guarantee a sufficient agreement level among DMs before making a decision: it may occur that the solution found is not accepted by some DMs who consider that their individual concerns have not been addressed sufficiently [9, 12, 13]. In many real-life GDM settings, highly accepted group decisions become vital. Therefore, a consensus phase or *Consensus Reaching Process* (CRP) is introduced. A CRP is a process of discussion and modification of preferences by DMs, aimed at bringing them closer to each other, towards a collective opinion deemed as acceptable by the whole group [12]. The classical view of consensus as full agreement (unanimity) evolved towards more flexible interpretations, establishing that consensus is measured as a level of (partial) agreement, which indicates how far the opinions of DMs are from unanimity [13].

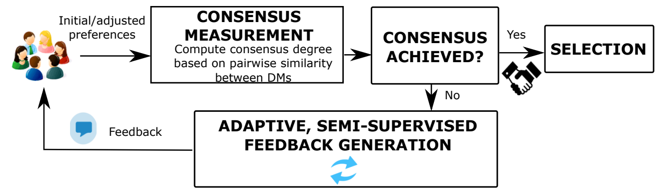

Fig. 2. Consensus model scheme

The process to reach consensus is iterative, dynamic and classically coordinated by a human figure known as *moderator*, who is responsible for supervising and assisting DMs throughout the process. A general CRP scheme followed by most consensus models proposed in the literature [13] is shown in Figure 1. Its phases are described below:

1) *Consensus Measurement*: Preferences of DMs are gathered to compute the current level of agreement in the group, using a consensus measure.
2) *Consensus Control*: The consensus degree is compared with a threshold μ defined a priori. If the consensus degree exceeds this threshold, the group moves on to the selection process; otherwise, another round of discussion is required.
3) *Consensus Progress*: A procedure is applied to increase the level of agreement in the following CRP round. Some consensus models involve a *feedback generation* process to do this, such that DMs are advised to modify their farthest preferences from consensus [9, 12], whilst other models implement approaches that update information (e.g. assessments of experts or their importance weights) to find a consensus collective opinion [21, 22].

III. Consensus Model for MCLGDM with CLEs

This section introduces a consensus model for MCLGDM problems in which DMs use CLEs to evaluate the alternatives. Unlike previous consensus approaches for GDM settings with uncertain linguistic preferences, this contribution introduces an adaptive, semi-supervised feedback mechanism to reduce both the uncertainty caused by DMs hesitancy, and the substantial cost required to revise a considerable number of DMs' preferences. We start by overviewing the phases of the consensus model. Subsequently, its novel feedback mechanism is described in detail.

A. Consensus Model Overview

Figure 2 shows a schema of the consensus model, whose phases are described below.

1) *Gathering Preferences*: Each DM constructs and provides an evaluation matrix, $P_i = (p_i^{jk})_{n \times l}$, which represents her/his preferences over alternatives in X, based on criteria in C. Each assessment p_i^{jk} is a CLE on S.
2) *Computing Consensus Degree*: The degree of consensus in the group is measured as a numerical value in the unit interval, $cr \in [0, 1]$, such that the closer cr is to one, the closer DMs are from unanimous agreement.In this consensus model, the procedure to obtain cr is

based on calculating similarities between pairs of DMs' preferences [13]. It consists of the following steps:

i. Calculate, for each pair of DMs e_i, e_u, $i < u$, a *similarity matrix* $SM_{iu} = (sm_{iu}^{jk})_{n\times l}$. Given m DMs, a total of $m(m-1)/2$ similarity matrices are calculated. Each element $sm_{iu}^{jk} = sim(p_i^{jk}, p_u^{jk}) \in [0,1]$ represents the degree to which the opinions of e_i and e_u on the pair (x_j, c_k) are similar. Let $H_i^{jk} = \mathcal{E}(p_i^{jk})$ and $H_u^{jk} = \mathcal{E}(p_u^{jk})$ denote the equivalent HFLTSs to the CLEs p_i^{lk}, p_u^{lk}, respectively. Furthermore, let $\pi_i^{jk} = \{\langle s_h, \beta_h\rangle\}$ be the possibility distribution associated to H_i^{lk}, such that $\beta_h = 1/(U-L+1)$ for each $s_h \in H_i^{jk}$. The similarity between assessments p_i^{jk} and p_u^{jk} is calculated based on π_i^{jk}, π_u^{jk} as [23]:

$$sm_{iu}^{jk} = 1-d(\pi_i^{jk},\pi_u^{jk}) = 1-\frac{1}{2}\sum_{h=0}^{g-1}|\beta_{hi}^{jk}-\beta_{hu}^{jk}| \quad (3)$$

For computational convenience, if $\beta_{hi}^{jk} \notin \pi_i^{jk}$, then we assume in Eq. (3) that $\beta_{hi}^{jk} = 0$.

ii. Using an aggregation function ϕ, similarity matrices are fused into a *consensus matrix* $CM = (cm^{jk})_{n\times l}$:

$$cm^{jk} = \phi(sm_{12}^{jk}, \ldots, sm_{1m}^{jk}, \ldots, sm_{(m-1)m}^{jk}) \quad (4)$$

Each cm^{jk} represents the group level of consensus between DMs assessments on (x_j, c_k).

iii. For each row of the consensus matrix CM, consensus degrees are aggregated across criteria to obtain a consensus degree ca^j on each alternative x_j:

$$ca^j = \frac{\sum_{k=1}^{l} cm^{jk}}{l} \quad (5)$$

iv. The overall consensus degree, cr, is finally determined by successive aggregation of alternative-level consensus degrees, as follows:

$$cr = \frac{\sum_{j=1}^{n} ca^j}{n} \quad (6)$$

3) *Consensus Control*: The consensus degree is compared with a threshold agreement level $\mu \in]0,1]$ established a priori. If $cr \geq \mu$ the CRP ends having reached the desired consensus and the group proceeds to the alternatives selection process. Otherwise, if the number of CRP rounds conducted so far does not exceed a limit $Maxround \in \mathbb{N}$, the group moves on to the feedback generation phase.

4) *Feedback Generation*: This phase aims at bringing DMs preferences closer to each other to increase consensus. Traditionally, a human moderator has been the entity responsible for coordinating these actions for consensus building. Nonetheless, recent developments in Consensus Support Systems have enabled the assistance to groups in this process [13]. The feedback generation phase consists of five sub-steps:

i. *Compute Collective Preference*: The Hesitant Fuzzy Linguistic Weighted Average (HFLWA) operator [8] is used to aggregate individual DMs preferences, $P_1, \ldots, P_m$, into a collective preference matrix. Firstly, the m associated HFLTSs for each m are aggregated at alternative-criterion level, (x_j, c_k):

$$\begin{aligned}\pi_c^{jk} &= HFLWA_W(H_1^{jk}, \ldots, H_m^{jk}) \\ &= DAA(\pi^1, \pi^2, \ldots, \pi^{\prod_{i=1}^m |H_i^{jk}|})\end{aligned} \quad (7)$$

with $W = [w_1 \ \ldots \ w_m]$, $w_i \in [0,1]$, $\sum_i w_i = 1$ a weight vector that assigns importance degrees to DMs[3]. The underlying DAA operator [8] is an arithmetic average of possibility distributions π^z, with each $\pi^z = \{\langle s_h, \beta_h\rangle\}$, $0 < \beta_h \leq 1$, $\sum_h \beta_h = 1$, a possibility distribution over S that results from combining *one* linguistic term $s^i \in H_i^{jk}$ of each HFLTS separately. Given m aggregation inputs $H_1^{jk}, \ldots, H_m^{jk}$, the resulting number of distributions equals $\prod_{i=1}^m |H_i^{jk}|$. Each π^z is calculated as follows:

$$\pi^z = \begin{cases} \{\langle s_{\lfloor\theta\rfloor}, 1-\{\theta\}\rangle, \langle s_{\lfloor\theta\rfloor+1}, \{\theta\}\rangle\} & \text{if } \{\theta\} \neq 0, \\ \{\langle s_\theta, 1\rangle\} & \text{if } \{\theta\} = 0. \end{cases} \quad (8)$$

Importantly, $\theta = \sum_{i=1}^m w_i \cdot index(s^i)$ is a real value in the $[0,g]$ interval, with $index(s^i) = h \in \{0,\ldots,g\}$; and $\lfloor\theta\rfloor$, $\{\theta\}$ denote the integer and fractional parts of θ, respectively. An example that illustrates the use of the HFLWA operator to aggregate individual assessments is provided at the end of this subsection.

ii. *Convert collective assessments into HFLTSs*: The result of the HFLWA aggregation, π_c^{jk}, is a possibility distribution across *consecutive* linguistic terms in S, as shown in [8]. Notwithstanding, our interest focuses on providing DMs with understandable feedback information in the form of CLEs, as well as reducing the presence of highly uncertain assessments, particularly as the degree of consensus increases. In accordance with this, a *uniform HFLTS* H_c^{jk} is derived from π_c^{jk} as follows:

$$H_c^{jk} = \{s_h \in S | \beta_h \geq \epsilon; \beta_h \in \pi_c^{jk}\} \quad (9)$$

with $\epsilon = 1/|\pi_c^{jk}|$. In other words, a collective assessment is represented as an HFLTS containing the terms $s_h \in S$ with possibility degrees $\beta_h \in \pi_c^{jk}$ greater than a possibility threshold ϵ, inversely proportional to $|\pi_c^{jk}|$. As a result, the HFLTS-based collective preference $P_c = (H_c^{jk})_{n\times l}$ is obtained.

iii. *Compute Proximity Matrices*: A proximity matrix $PM_i = (pm_i^{jk})_{n\times l}$ indicating the similarity between each DM and P_c, is obtained for each $e_i \in E$. Thus, each proximity value $pm_i^{jk} = sim(H_i^{jk}, H_c^{jk}) \in [0,1]$ is calculated using a similarity measure between HFLTSs, using Eq. (3).

iv. *Identify Farthest Preferences from Consensus*: Prox-

[3]In many group decision scenarios, DMs are assigned distinct weights w_i based on their level of expertise, background, etc. If all DMs are regarded equally important, then $w_i = 1/m$, $\forall i$.

Algorithm 1 Direction rules for DM e_i to increase assessment $p_i^{jk} = \Lambda(\{s_L, \dots, s_U\})$

if ($|H_i^{jk}|$ == 1) **then**
  if $(T(H_i^{jk}) - g/2)(T(H_c^{jk}) - g/2) < 0$ **then**
    Advise $\Lambda(\{s_h\}) \rightarrow \Lambda(\{s_{h+1}\})$
  else
    Automatic Update $\Lambda(\{s_h\}) \rightarrow \Lambda(\{s_{h+1}\})$
  end if
else
  if $((T(H_i^{jk}) - g/2)(T(H_c^{jk}) - g/2) < 0)$ **then**
    if $|H_i^{jk}| \leq 3$ **then**
      Advise $\Lambda(\{s_L, \dots, s_U\}) \rightarrow \Lambda(\{s_{L+1}, \dots, s_U\})$
    else
      Advise $\Lambda(\{s_L, \dots, s_U\}) \rightarrow \Lambda(\{s_{L+2}, \dots, s_{U-1}\})$
    end if
  else
    if $|H_i^{jk}| \leq 3$ **then**
      Automatic Update $\Lambda(\{s_L, \dots, s_U\}) \rightarrow \Lambda(\{s_{L+1}, \dots, s_U\})$
    else
      Automatic Update $\Lambda(\{s_L, \dots, s_U\}) \rightarrow \Lambda(\{s_{L+2}, \dots, s_{U-1}\})$
    end if
  end if
end if

imity values are analyzed to identify which preferences are farthest from the group opinion and should be modified by (some of the) DMs. Alternative-criterion pairs (x_j, c_k) whose consensus degrees cm^{jk} and ca^j are not sufficient, are firstly identified:

$$CC = \{(x_j, c_k) | ca^j < cr \wedge cm^{jk} < cr\} \qquad (10)$$

DMs who should modify their opinion on each pair in CC, are subsequently identified predicated on an average proximity value for that pair, $\overline{pm}^{jk}$, calculated using an averaging operator λ:

$$\overline{pm}^{jk} = \lambda(pm_1^{jk}, \dots, pm_m^{jk}) \qquad (11)$$

As a result, those DMs' assessments accomplishing $pm_i^{jk} < \overline{pm}^{jk}$, are fed into a set of adaptive, semi-supervised direction rules.

v. *Generate Advice upon Direction Rules*: A number of direction rules dependent on the current level of consensus achieved, are applied to (i) suggest DMs how to modify their identified assessments, and (ii) decide whether proposed changes are not significant enough to require human supervision and apply them automatically. Section III-B describes in further detail the direction rule approach, which plays a central role in our proposed adaptive semi-supervised feedback mechanism.

Example 2. (**Preference aggregation**) Consider $S = \{N, VL, L, M, H, VH, A\}$ (with N:Null and A:Absolute), and three DMs assessments such that $H_1^{jk} = \{VH, A\}$, $H_2^{jk} = \{A\}$, $H_3^{jk} = \{VL, L\}$, with $w_i = 1/3, \forall e_i$. Eq. (8) is firstly applied to obtain $\prod_{i=1}^{3} |H_i^{jk}| = 4$ possibility distributions:

$\pi^1 = \{\langle H, 1\rangle\}$ $\qquad$ $\pi^2 = \{\langle H, 0.67\rangle, \langle VH, 0.33\rangle\}$,
$\pi^3 = \{\langle H, 0.67\rangle, \langle VH, 0.33\rangle\}$ $\qquad$ $\pi^4 = \{\langle H, 0.33\rangle, \langle VH, 0.67\rangle\}$

For instance π^4 stems from combining $\{A\} \in H_1^{jk}$, $\{A\} \in H_2^{jk}$ and $\{L\} \in H_3^{jk}$ into θ (based on weights of DMs), as follows: $\theta = 1/3 \cdot 6 + 1/3 \cdot 6 + 1/3 \cdot 2 = 4.67$. Finally, by applying Eq. (7) we have $\pi_c^{jk} = \{\langle H, 0.66\rangle, \langle VH, 0.34\rangle\}$. $\square$

B. Adaptive, Semi-Supervised Feedback Mechanism

The objective of the proposed direction rules approach is threefold: (i) to decide whether delivering bespoke advice for DMs or automatically updating their farthest assessments from consensus, based on the CRP status and the characteristics of such assessments; (ii) to identify highly uncertain assessments and aid DMs in reducing hesitancy based on their position with respect to the consensus opinion; and (iii) to transparently operate with HFLTS information, yet providing highly understandable feedback in the form of CLEs. Thus, this approach presents the following characteristics:

- *Semi-supervised*: when applying a direction rule does not involve a significant shift in the DM opinion, p_i^{jk} is automatically updated without requesting human supervision. This allows to reduce the cost of the CRP and making the consensus model more scalable to accommodate large-group decision situations.
- *Adaptive*: the level of uncertainty in each identified assessment is also analysed to define an adaptive feedback generation mechanism that contributes to reducing such uncertainty.
- *User-friendly*: feedback are returned to DMs in the form of suggested CLEs (e.g. *move from "between Very Low and Low" to "Low"*) to facilitate their understanding.

Algorithm 1 shows the direction rule-based feedback generation procedure for the cases when an identified assessment p_i^{jk} (by Eq. (10)) holds $H_i^{jk} < H_c^{jk}$, i.e. e_i's preference towards x_j under c_k should *increase*. Analogously, the feedback generation process for $H_i^{jk} > H_c^{jk}$ (*decrease* preference)

Algorithm 2 Direction rules for DM e_i to decrease assessment $p_i^{jk} = \Lambda(\{s_L, \ldots, s_U\})$

if ($|H_i^{jk}|$ == 1) **then**
  if $(T(H_i^{jk}) - g/2)(T(H_c^{jk}) - g/2) < 0$ **then**
    Advise $\Lambda(\{s_h\}) \rightarrow \Lambda(\{s_{h-1}\})$
  else
    Automatic Update $\Lambda(\{s_h\}) \rightarrow \Lambda(\{s_{h-1}\})$
  end if
else
  if $((T(H_i^{jk}) - g/2)(T(H_c^{jk}) - g/2) < 0)$ **then**
    if $|H_i^{jk}| \leq 3$ **then**
      Advise $\Lambda(\{s_L, \ldots, s_U\}) \rightarrow \Lambda(\{s_L, \ldots, s_{U-1}\})$
    else
      Advise $\Lambda(\{s_L, \ldots, s_U\}) \rightarrow \Lambda(\{s_{L+1}, \ldots, s_{U-2}\})$
    end if
  else
    if $|H_i^{jk}| \leq 3$ **then**
      Automatic Update $\Lambda(\{s_L, \ldots, s_U\}) \rightarrow \Lambda(\{s_L, \ldots, s_{U-1}\})$
    else
      Automatic Update $\Lambda(\{s_L, \ldots, s_U\}) \rightarrow \Lambda(\{s_{L+1}, \ldots, s_{U-2}\})$
    end if
  end if
end if

is described in Algorithm 2. Both algorithms have the same rationale, which is explained for Algorithm 1 below.

- *The assessment p_i^{jk} is a single linguistic term*: The uncertainty degree of p_i^{jk} is minimal, and the resulting feedback preserves such low uncertainty, moving to the immediately next linguistic term in S. The feedback is automatically applied on p_i^{jk} if its score value and that of p_c^{jk} are both above or below the central linguistic term $s_{g/2}$, i.e. moving towards p_c^{jk} does not imply a significant change in the opinion of the DM.
- *The assessment p_i^{jk} is a CLE involving at least two consecutive linguistic terms*: In these cases, the uncertainty degree of p_i^{jk} can be reduced. For a moderate level of uncertainty ($|H_i^{jk}| \leq 3$), this is done by generating feedback in the form of a CLE whose score $T(H_i^{jk})$ not only becomes closer to the collective assessment score, $T(H_c^{jk})$, but also removes the upper and/or lower linguistic term from H_i^{jk} accordingly. This uncertainty reduction mechanism accentuates if the assessment is highly uncertain, namely if $|H_i^{jk}| > 3$. Again, the feedback is automatically applied on p_i^{jk} if its score value and that of p_c^{jk} are both above or below $s_{g/2}$.

The semi-supervision rules allow to reduce the degree of human supervision required over the course of the CRP, particularly when large groups of DMs take part in the decision problem, thus reducing the temporal cost of reaching consensus [15]. Figure 3 illustrates the direction rule mechanism.

IV. Experimental Study

To demonstrate the effectiveness and applicability of the proposed consensus model to handle large groups, a CRP is conducted to solve a real MCLGDM problem. A university school board intends to select the best candidate out of four $X = \{x_1, x_2, x_3, x_4\}$ to take up a lectureship. Besides the

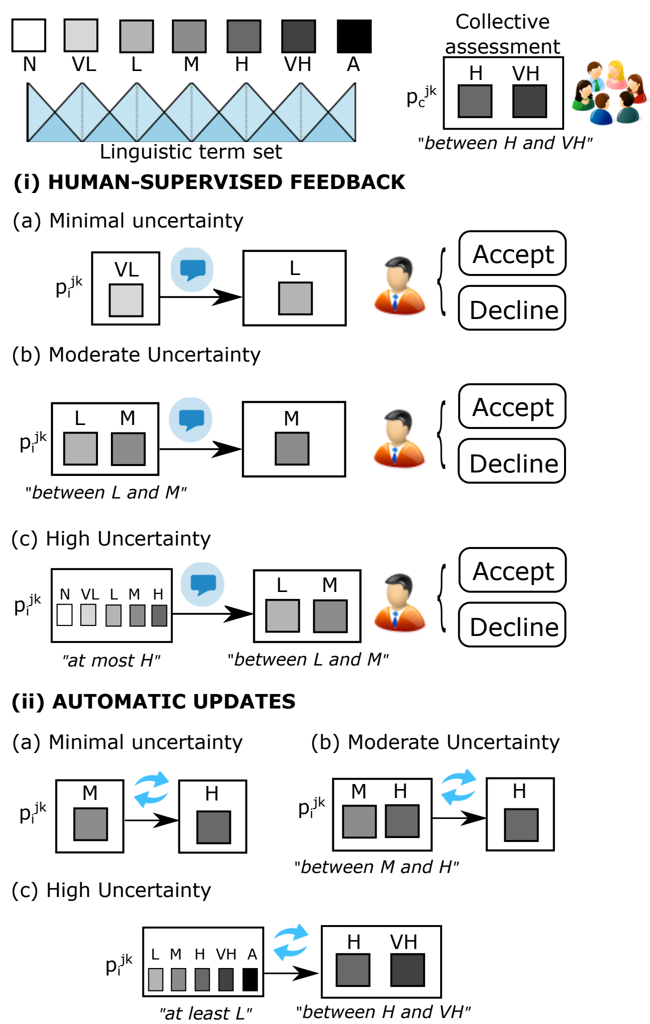

Fig. 3. Semi-supervised direction rule echanism

TABLE I
EVOLUTION OF CONSENSUS DEGREE AND NUMBER OF FEEDBACK SUPERVISIONS REQUIRED

	Semi-supervised model				Baseline	
t	cr	$\#All$	$\#Sup$	$\#Aut$	cr	$\#Sup$
1	0.737	18	**(7)**	11	0.729	19
2	0.749	25	15	10	0.723	**(12)**
3	0.768	17	**(14)**	3	0.731	15
4	0.782	22	**(9)**	13	0.741	23
5	0.82	8	**(3)**	5	0.765	26
6	0.839	7	**(2)**	5	0.788	12
7	**0.856**	14	**(4)**	10	0.815	16
8					0.842	19
9					**0.855**	4

formal interview, and as part of the recruitment process, the university invites 42 staff members, $E = \{e_1, e_2, \ldots, e_{42}\}$, to attend a 25-minute talk delivered by each candidate. The objective of the talk is to demonstrate and evaluate the ability shown by each candidate for $C = \{c_1 : teaching, c_2 : research\ vision, c_3 : public\ \ engagement\}$. Each DM accesses an online form (evaluation matrix). Due to the inherent vagueness and uncertainty that attendants (DMs) from different research backgrounds present, some of them are not confident enough in using a single linguistic term for some of their assessments. Therefore, DMs are allowed to use CLEs on the linguistic term set $S = \{N, VL, L, M, H, VH, A\}$ to assess each candidate and skill.

After the talks, an interactive discussion and consensus building session is held among participants, with the aid of the proposed methodology. To do this, the group is divided into two subgroups of 21 members each. For the first group, the CRP problem is firstly conducted by using the adaptive semi-supervised model. By contrast, the second group uses a *baseline* consensus model that does not incorporate semi-supervision rules. In both cases, the desired level of consensus is $\mu = 0.85$ and $Maxround = 10$. The consensus degree, cr, and the following three metrics are gathered per CRP round:

- $\#All$: Total number of assessments p_i^{jk} identified by the feedback mechanism.
- $\#Sup$: Number of assessments revised by the DM. In the baseline, fully supervised model, $\#Sup = \#All$.
- $\#Aut$: Number of assessments automatically updated.

Results are summarized in Table I. The first group (semi-supervised model) achieves a slightly higher convergence towards consensus, i.e. less discussion rounds are required to exceed μ. This goes in line with previous research results shown in [15]. Remarkably, by comparing $\#Sup$ between both methods (number of times an assessment requires manual human supervision), we can conclude that a significantly lower cost is invested by DMs in the first group in revising their preferences, in almost all CRP rounds (see grey-shaded cells in Table I). Interestingly, these cost differences become more evident at later consensus rounds, when cr gets closer to μ (DMs become closer to each other), because most "last-minute" adjustments still required on assessments become smaller.

We finally introduce a brief theoretical comparison of our consensus model with other similar approaches in the literature, namely three recent consensus models for decision making problems with uncertain linguistic preferential information. The selected literature works this comparison include: (i) the work by Wu and Xu in [24], (ii) the work by Dong et al. in [22] and (iii) the work by Xu and Wang in [25]. Despite all three being notable contributions, they present a number of differences among each other and with our proposed work, in terms of target decision framework and type of consensus approach adopted [13]. Table II summarises the key features of each work. In summary, our work differentiates from existing literature in:

- Being specifically conceived to deal with large groups of DMs [17], given its semi-supervised nature.
- Using CLEs not only for expressing preferences, but also for representing the feedback for DMs when human supervision is required, ensuring highly understandable and insightful feedback anytime during the CRP.
- Combing the benefits of (i) automatic adjustments of preferences to reduce the cost of reaching consensus [22], and (ii) the preservation of experts' sovereignty provided by human-supervised feedback mechanisms [24, 25], into our proposed semi-supervised approach.

V. CONCLUDING REMARKS

This contribution has presented a novel consensus framework for Multi-Criteria Large-Group Decision Making problems un-

[4]It does not require calculating proximity measures.
[5]Initial DMs preferences are preserved as much as possible.

TABLE II
COMPARISON WITH SIMILAR CONSENSUS MODELS

	Wu and Xu [24]	Dong et al. [22]	Xu and Wang [25]	Palomares et al.
Target decision framework	GDM	GDM	GDM	MCLGDM
Applied to large groups	No	No	No	Yes
Preference structure	Preference relation	Preference relation	Preference relation	Evaluation Matrix
Assessment format	HFLTS	HFLTS	CLE / Hesitant 2-tuple	CLE
Consistency-reaching process	Yes	No	Yes	No
Feedback mechanism	Yes (simplified[4])	No	Yes (minimum adjustment[5])	Yes (semi-supervised)
Automatic adjustments	No	Yes (optimization-based)	No	Yes (partially)
Type of feedback suggested	Increase/Decrease	(not required)	Increase/Decrease	A new CLE

der uncertainty, with preferences of decision makers expressed as linguistic comparative expressions. The consensus model uses an adaptive, semi-supervised feedback mechanism to deal with a substantial number of decision makers preferences and feedback, and reduce the high level of uncertainty in assessments while favoring consensus building. The proposed model is used to support a large-group decision problem involving academic recruitment, demonstrating its usefulness in reducing the effort required by decision makers to revise their preferences during the consensus reaching process.

Future works aim primarily at further studying the effects of incorporating uncertainty reduction approaches in the consensus feedback mechanism. We also plan to extend our work to (1) study consistency of comparative linguistic preferences; (2) deal with situations of incompleteness when some decision makers may not provide all assessments on alternatives and criteria; and (3) accommodate large groups in which different decision makers may use linguistic term sets of different granularities to assess alternatives.

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